## COMMUNICATION ENGINEERING

Mutual Information

## MUTUAL INFORMATION

$>$ Prior to the reception of a message the state of knowledge at the receiver about the transmitted signal $\mathrm{x}_{\mathrm{j}}$ (channel input) is the probability $\mathrm{p}\left(\mathrm{x}_{\mathrm{j}}\right)$
$>$ After the reception and selection of symbol $\mathrm{y}_{\mathrm{k}}$ (channel output) the state of knowledge about $\mathrm{x}_{\mathrm{j}}$ is the conditional probability $\mathrm{p}\left(\mathrm{x}_{\mathrm{j}} \mid \mathrm{y}_{\mathrm{k}}\right)$
$>$ Before $\mathrm{y}_{\mathrm{k}}$ is received uncertainty is $-\log \mathrm{p}\left(\mathrm{x}_{\mathrm{j}}\right)$
$>$ After $\mathrm{y}_{\mathrm{k}}$ is received uncertainty is $-\log \mathrm{p}\left(\mathrm{x}_{\mathrm{j}} \mid \mathrm{y}_{\mathrm{k}}\right)$
$>$ The information gained about $\mathrm{x}_{\mathrm{j}}$ by the reception of $\mathrm{y}_{\mathrm{k}}$ is the net reduction in its uncertainty known as MUTUAL INFORMATION $\mathrm{I}\left(\mathrm{x}_{\mathrm{j}}, \mathrm{y}_{\mathrm{k}}\right)$ ie. uncertainty about the channel input that is resolved by observing channel out put

## MUTUAL INFORMATION

$\Rightarrow \mathrm{I}\left(\mathrm{x}_{\mathrm{j}}, \mathrm{y}_{\mathrm{k}}\right)=$ initial uncertainty -final uncertainty

$$
=-\log \mathrm{p}\left(\mathrm{x}_{\mathrm{j}}\right)-\left(-\log \mathrm{p}\left(\mathrm{x}_{\mathrm{j}} \mid \mathrm{y}_{\mathrm{k}}\right)\right)
$$

$$
\mathrm{I}\left(\mathrm{x}_{\mathrm{j}}, \mathrm{y}_{\mathrm{k}}\right)=\log \left(\mathrm{p}\left(\mathrm{x}_{\mathrm{j}} \mid \mathrm{y}_{\mathrm{k}}\right) / \mathrm{p}\left(\mathrm{x}_{\mathrm{j}}\right)\right.
$$

$$
\text { Also, } \quad=\log \left(\mathrm{p}\left(\mathrm{y}_{\mathrm{k}} \mid \mathrm{x}_{\mathrm{j}}\right) / \mathrm{p}\left(\mathrm{y}_{\mathrm{k}}\right)\right.
$$

$$
\mathrm{I}\left(\mathrm{x}_{\mathrm{j}}, \mathrm{y}_{\mathrm{k}}\right)=\mathrm{I}\left(\mathrm{y}_{\mathrm{k}}, \mathrm{x}_{\mathrm{j}}\right)
$$

$>$ Average of mutual information is the entropy corresponding to mutual information

$$
\mathrm{I}(\mathrm{X} ; \mathrm{Y})=\overline{\mathrm{I}\left(\mathrm{x}_{\mathrm{j}}, \mathrm{y}_{\mathrm{k}}\right)}
$$

$\Rightarrow \mathrm{I}(\mathrm{X} ; \mathrm{Y})=\mathrm{H}(\mathrm{X})-\mathrm{H}(\mathrm{X} \mid \mathrm{Y})$

$$
\begin{aligned}
& =\mathrm{H}(\mathrm{Y})-\mathrm{H}(\mathrm{Y} \mid \mathrm{X}) \\
& =\mathrm{H}(\mathrm{X})+\mathrm{H}(\mathrm{Y})-\mathrm{H}(\mathrm{X}, \mathrm{Y})
\end{aligned}
$$

$>$ It is a measure of information transferred through the channel also called transferred information of the channel or trans information of the channel

## Problems

1. A discrete source transmits messages $\mathrm{x}_{1}, \mathrm{x}_{2}, \mathrm{x}_{3}$ with probabilities $0.3,0.4$ and 0.4 The source is connected to the channel given in figure. Calculate all the associated entropies and mutual information.


## Problems

2.Find the mutual information of the channel


## THANHOL

